Renaissance High School

Mathematics Department

Algebra 2 Summer Packet

Completion of summer packets is mandatory

Algebra 2 Essential Vocabulary (please define each world and give an example)

1) Whole numbers
2) Integers
3) Rational numbers
4) Irrational numbers
5) Variable
6) Like terms
7) Equation
8) Absolute value
9) Relation
10) Domain
11) Range
12) Function
    a. Linear Function
    b. Quadratic Function
    c. Polynomial Function
    d. Radical Function
    e. Exponential Function
    f. Natural Base Function
    g. Logarithmic Function
    h. Rational Function
    i. Trigonometric Function
13) X-intercept and Y-intercept
14) Parallel and perpendicular lines
15) Piecewise function
16) System of linear equations
17) System of linear inequalities
18) Matrix
19) Parabola
20) Vertex of a parabola
21) Polynomial
    a. Monomial
    b. Binomial
    c. Trinomial
22) Factoring
23) Zero product property
    a. Zero
    b. Solution

c. Root

24) Square root
25) Radicand
26) Radical
27) Rationalizing the denominator
28) Imaginary unit, i
29) Complex number
30) Imaginary number
31) Pure imaginary number
32) Quadratic formula
33) Discriminant
34) Leading coefficient
35) Constant term
36) Degree of a polynomial function
37) End behavior
38) Remainder theorem
39) Factor theorem
40) Fundamental theorem of Algebra
41) Repeated solution
42) Index
43) Like radicals
44) Function composition
45) Inverse function
46) Extraneous solution
47) Asymptote
48) Growth factor
49) Decay factor
50) Natural base, e (Euler's number)
51) Logarithm
52) Common logarithm
53) Natural logarithm
54) Change-of-base formula
55) Complex fraction
56) Cross multiplying
57) Distance formula
58) Midpoint formula
59) Sine
60) Cosine
61) Tangent
62) Secant

63) Cotangent
64) Radian
65) Inverse sine
66) Inverse cosine
67) Inver tangent
68) Law of sines
69) Law of cosines
70) Periodic function
71) Cycle
72) Period
73) Amplitude
74) Frequency
75) Trigonometric identities

Real Numbers and Algebraic Expressions

1) Give an example of a whole number that is not a natural number. Explain why this number fits the description.

2) Give an example of a rational number that is not an integer. Explain why this number fits the description.

3) Give an example of a number that is a rational number, an integer, and a real number. Explain why this number fits the description.

4) Can a real number be both rational and irrational? Explain your answer.

5) Are first getting undressed and then taking a shower commutative?

6) The algebraic expression $1527x + 31{,}290$ approximates average yearly earnings for elementary and secondary teachers in the United States x years after 1990. Evaluate the expression for $x = 10$. Describe what the answer means in practical terms.

7) The optimum heart rate is the rate that a person should achieve during exercise for the exercise to be most beneficial. The algebraic expression $0.6(220 - a)$ describes a person's optimum heart rate, in beats per minute, where *a* represents the age of the person.
    a. Use the distributive property to rewrite the algebraic expression
    b. Use each form of the algebraic expression to determine the optimum heart rate for a 20-year-old runner.

8) Simplify
    a. $5(3x + 4) - 4$
    b. $4(2y - 6) + 3(5y + 10)$
    c. $6 - 5[8 - (2y - 4)]$

Exponents and Scientific Notation

9) Use 10^{12} for one trillion and 2.8×10^{8} for the U.S. population in 2000 to solve the following exercise:
In 2000, the government collected approximately $1.9 trillion in taxes. What was the per capita tax burden, or the amount that each U.S. Citizen pain in taxes? Round to the nearest hundred dollars.

10) The mass of one oxygen molecule is 5.3×10^{-23} gram. Find the mass of 20,000 molecules of oxygen. Express the answer in scientific notation.

11) Why is $(-3x^{2})(2x^{-5})$ not simplified. What must be done to simplify the expression?

12) Explain the product rule for exponents. Use $2^{3} \cdot 2^{5}$ in your explanation.

13) Explain the power rule for exponents. Use $(3^{2})^{4}$ in your explanation.

14) Simplify

a. $\left(\frac{-15a^4b^2}{5a^{10}b^{-3}}\right)^3$

b. $\left(\frac{-30a^{14}b^8}{10a^{17}b^{-2}}\right)^2$

c. $\left(\frac{3a^{-5}b^8}{12a^3b^{-4}}\right)^0$

d. $(-5x^4y)(-6x^7y^{11})$

Radicals and Rational Exponents

(Reference: http://www.purplemath.com/modules/exponent5.htm)

15) The algebraic expression $2\sqrt{5L}$ is used to estimate the speed of a car prior to an accident, in miles per hour, based on the length of its skid marks, L, in feet. Find the speed of a car that left skid marks 40 feet long, and write the answer in simplified radical form.

16) The algebraic expression $152a^{-1/5}$ describes the percentage of U.S. taxpayers who are *a* years old who file early. Evaluate the algebraic expression for $a = 32$. Describe what the answer means in practical terms.

17) Use the product rule of radicals to simplify

a. $\sqrt{500}$
b. $\sqrt{6x}\cdot\sqrt{3x}$
c. $\sqrt{y^3}$
d. $\sqrt{2x^2}\cdot\sqrt{6x}$
e. $\sqrt{27}$

18) Use the quotient rule of radicals to simplify

a. $\sqrt{\frac{1}{81}}$
b. $\sqrt{\frac{121}{9}}$
c. $\frac{\sqrt{48x^3}}{\sqrt{3x}}$
d. $\frac{\sqrt{500x^3}}{\sqrt{10x^{-1}}}$
e. $\frac{\sqrt{200x^3}}{\sqrt{10x^{-1}}}$

19) Rationalize the denominator

a. $\frac{1}{\sqrt{7}}$

b. $\frac{2}{\sqrt{10}}$

c. $\frac{\sqrt{2}}{\sqrt{5}}$

d. $\frac{\sqrt{7}}{\sqrt{3}}$

e. $\frac{3}{3+\sqrt{7}}$

20) Simplify or indicate that the root is not a real number

a. $\sqrt[3]{125}$

b. $\sqrt[4]{-16}$

c. $\sqrt[3]{-8}$

d. $\sqrt[5]{-\frac{1}{32}}$

e. $\sqrt[3]{8}$

f. $\sqrt[3]{9}\cdot\sqrt[3]{6}$

21) Add or subtract like radicals whenever possible

a. $4\sqrt[5]{2}+3\sqrt[5]{2}$

b. $\sqrt[3]{24xy^3}-y\sqrt[3]{81x}$

c. $\sqrt{2}+\sqrt[3]{8}$

d. $\sqrt{7}-3\sqrt{7}$

22) Simplify using properties of exponents

a. $(7x^{1/3})(2x^{1/4})$

b. $(3x^{2/3})(4x^{3/4})$

c. $(x^{4/5})^5$

d. $\frac{(2y^{1/5})^4}{y^{3/10}}$

e. $(25x^4y^6)^{1/2}$

f. $\sqrt[3]{6}\cdot\sqrt[7]{6}$

Polynomials

23) Write the expression in standard form if it is a polynomial

a. $2x + 3x^2 - 5$

b. $2x + 3x^{-1} - 5$

c. $x^2 - x^3 + x^4 - 5$

d. $\dfrac{2x+3}{x}$

24) Perform the indicated operations

a. $(-6x^3 + 5x^2 - 8x + 9) + (17x^3 + 2x^2 - 4x - 13)$

b. $(17x^3 - 5x^2 + 4x - 3) - (5x^3 - 9x^2 - 8x + 11)$

c. $(5x^2 - 7x - 8) + (2x^2 - 3x + 7) - (x^2 - 4x - 3)$

d. $(8x^2 + 7x - 5) - (3x^2 - 4x) - (-6x^3 - 5x^2 + 3)$

e. $(x + 1)(x^2 - x + 1)$

f. $(2x - 3)(x^2 - 3x + 5)$

g. $(x + 7)(x + 3)$

h. $(2x - 1)(x^2 - 4x + 3)$

i. $(4 - 3x)(4 + 3x)$

j. $(5x^2 - 3)^2$

k. $(2x - 3)^3$

l. $(x + 5y)(7x + 3y)$

m. $(x + y)(x^2 - xy + y^2)$

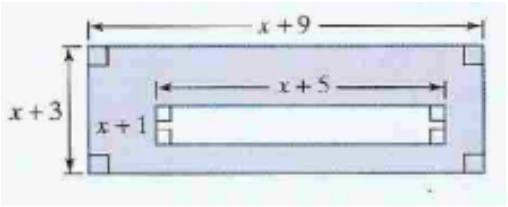
n. $(x - y)(x^2 + xy + y^2)$

25) The polynomial $0.018x^2 - 0.757x + 9.047$ describes the amount, in thousands of dollars, that a person earning x thousand dollars a year feels underpaid. Evaluate the polynomial for x = 40. Describe what the answer means in practical terms.

26) The polynomial $104.5x^2 - 1501.5x + 6016$ describes the death rate per year, per 100,000 men, for men averaging x hours of sleep each night. Evaluate the polynomial for x = 10. Describe what the answer means in practical terms.

27) The polynomial function $f(x) = -0.87x^3 + 0.35x^2 + 81.62x + 7684.94$ models the number of thefts, f(x), in thousands, in the United States x years after 1987. Will this function be useful in modeling the number of thefts over an extended period of time? Explain your answer.

28) Write a polynomial that represents the area of the large rectangle. Write a polynomial that represents the area of the small, unshaded rectangle. Write a polynomial that represents the area of the shaded region.



Factoring Polynomials

29) Factor out the greatest common factor
   a. $18x + 27$
   b. $6x^4 - 18x^3 + 12x^2$
   c. $9x^5 - 18x^3 + 27x^2$

30) Factor by grouping
   a. $x^3 - 2x^2 + 5x - 10$
   b. $x^3 - 3x^2 + 4x - 12$
   c. $3x^3 - 2x^2 - 6x + 4$
   d. $x^3 - x^2 - 5x - 5$

31) Factor each trinomial
   a. $x^2 + 5x + 6$
   b. $x^2 + 8x + 15$
   c. $x^2 - 14x + 45$
   d. $2x^2 + 5x - 3$
   e. $3x^2 - 25x - 28$
   f. $4x^2 + 16x + 15$

32) Factor the difference of two squares
   a. $x^2 - 100$
   b. $36x^2 - 49$
   c. $64x^2 - 81$
   d. $x^2 - 144$

33) Factor any perfect square trinomials, or state that the polynomial is prime
   a. $x^2 + 2x + 1$
   b. $x^2 + 4x + 4$
   c. $25x^2 + 10x + 1$
   d. $x^2 - 10x + 25$

34) Factor using the sum or difference of cubes (Reference: http://www.purplemath.com/modules/specfact2.htm)

   a. $x^3 + 27$
   b. $x^3 - 64$
   c. $64x^3 + 27$
   d. $8x^3 + 125$
   e. $27x^3 - 1$

35) The polynomial $8x^2 + 20x + 2488$ describes the number, in thousands, of high school graduates in the United States x years after 1993.

a. According to this polynomial, how many students graduated in 2003?
b. Factor the polynomial.
c. Use the factored form of the polynomial in part b to find the number of high school graduates in 2003. Do you get the same answer as you did in part a? If so, does this prove that your factorization is correct? Explain.

Linear Equations

36) Solve and check the linear equations

a. $7x - 5 = 72$
b. $5x - (2x - 10) = 35$
c. $13x + 14 = 12x - 5$
d. $5x - (2x + 2) = x + (3x - 5)$
e. $45 - [\,4 - 2y - 4(y + 7)] = -4(1 + 3y) - [4 - 3(y + 2) - 2(2y - 5)]$

37) Solve each equation with constants in the denominator

a. $\frac{x}{3} = \frac{x}{2} - 2$

b. $\frac{x}{2} = \frac{3x}{4} + 5$

c. $2x - \frac{2x}{7} = \frac{x}{2} + \frac{17}{2}$

d. $5 + \frac{x-2}{3} = \frac{x+3}{8}$

38) Solve each equation for the specified variable

a. A = lw, for w
b. $A = \frac{bh}{2}$, for b
c. I = Prt for P
d. $A = \frac{1}{2}h(a+b)$, for a
e. P = C + MC for M
f. $S = \frac{C}{1-r}$, for r

39) Solve each absolute value equation

a. $|x - 2| = 7$
b. $2|3x - 2| = 14$
c. $|x + 1| + 6 = 2$

40) The formula $\frac{W}{2} - 3H = 53$ describes the recommended weight W, in pounds, for a male, where H represents the man's height, in inches, over 5 feet. What is the recommended weight for a man who is 6 feet, 3 inches tall?

41) What does it mean to solve an equation?

Quadratic Equations

42) The formula $N = 23.4x^2 - 259.1x + 815.8$ shows the number of convictions of police officers throughout the US from 1994 to 2000, where N is the number of police officers convicted of felonies x years after 1990. In what year will 1000 police officers be convicted of felonies?

43) The function $f(x) = -0.03x^2 + 0.14x + 1.43$ models U.S. movie attendance, $f(x)$, in billions of tickets sold, x years after 2000.
   a. According to this function, how many billions of movie tickets were sold in 2005?
   b. According to this function, in which year was movie attendance at a maximum? Round to the nearest year. What does the function give for the billions of tickets sold for that year?

44) The length of a rectangle is 6 inches more than its width. The area of the rectangle is 91 square inches. Find the dimensions of the rectangle by creating an equation for the area and finding the solutions of the equation.

45) Find three consecutive integers such that four times the sum of all three is 2 times the product of the larger two.

46) The hypotenuse of a right triangle is 6 more than the shorter leg. The longer leg is three more than the shorter leg. Find the length of the shorter leg.

47) Solve by factoring
   a. $x^2 - 3x - 10 = 0$
   b. $x^2 = 8x - 15$
   c. $x^2 - 13x + 36 = -$
   d. $5x^2 - 20x = 0$
   e. $16x(x - 2) = 8x - 25$
   f. $10x - 1 = (2x + 1)^2$

48) Solve using the square root method
   a. $3x^2 = 27$
   b. $5x^2 + 1 = 51$
   c. $(3x + 2)^2 = 9$
   d. $(3x - 4)^2 = 8$

49) Determine the constant that should be added to the binomial to make a perfect square trinomial
   a. $x^2 + 12x$
   b. $x^2 + 16x$
   c. $x^2 + 5x$
   d. $x^2 - 9x$
   e. $x^2 - \frac{2}{3}x$

50) Solve using the quadratic formula
   a. $3x^2 - 4x - 4 = 0$
   b. $5x^2 + x - 2 = 0$
   c. $3x^2 = 6x - 1$
   d. $4x^2 = 2x + 7$

51) Find the discriminant. What is the number and type of solutions each quadratic has?
   a. $x^2 - 4x - 5 = 0$
   b. $4x^2 - 2x + 3 = 0$
   c. $3x^2 = 2x - 1$
   d. $3x^2 + 4x - 2 = 0$
   e. $x^2 - 3x - 7 = 0$

Systems of Equations

52) From 1840 to 1990 the percent of the labor force in farming and non-farming occupations can be modeled by the following equations where t is the number of years since 1840.

$y = -0.48t + 62.7$

$y = 0.48t - 32.9$

In what year was the labor force split equally between farming and non-farming occupations? Round your answer to the nearest year.

53) You and nine friends have decided to take a few days to go camping in Pennsylvania's Allegheny National Forest. You are planning a recreational budget of $50 per day for activities. Your group is considering the amusement park on the outskirts of the forest. The bumper cars cost $3.00 per hour and two people can ride in each car. Another choice is the water slide. The cost is $8.00 per person. Write an equation that shows how many bumper car tickets b and water slide passes w you would need to purchase. Write an equation that expresses how much the bumper car tickets and water slide passes would cost and remain within the confines of your recreational budget. According to the equations, how many bumper cars and water slide passes can you purchase?

54) In this exercise, let x represent the number of years after 1985 and let y represent the percentage of Americans in one of the groups shown who used cigarettes. Use the data points (0, 38) and (20, 27.3) to find the slope-intercept equation of the line that models the percentage of African Americans who used cigarettes, y, x years after 1985. Round the value of the slope m to two decimal places. Use the data points (0, 40) and (20, 24.2) to find the slope-intercept equation of the line that models the percentage of Hispanics who used cigarettes, y, x years after 1985. Use the models from parts (a) and (b) to find the year during which cigarette use was the same for African Americans and Hispanics. What percentage of each group used cigarettes during that year?

Exponential Growth and Decay

55) The world population in 2000 was approximately 6.08 billion. The annual rate of increase was about 1.26%
   a. Find the growth factor for the world population _____
   b. Write a function to model the world population _____
   c. Let your variable be the number of years after 2000. Use the model to find the world population in 2020. _____

56) Bacteria can multiply at an alarming rate when each bacteria splits into two new cells, thus doubling. If we start with only one bacterium, which can double every hour, how many bacteria will we have by the end of one day (show work)?

57) You buy a commemorative coin for $110. Each year *t,* the value V of the coin increases by 4%.
   a. Write an exponential growth model that describes the situation.
   b. Find the value of the coin after three years

58) You purchase an antique table for $525. Each year *t*, the value V of the table increases by 5%.
   a. Write an exponential growth model that describes the situation.
   b. Find the value of the table after ten years

59) You buy a new MacBook for $2100. The value of the computer decreases by about 50% annually.
   a. Write an exponential decay model for the value of the computer.
   b. Use the model to estimate the value after 2 years.

60) You buy a new car for $22,000. The value of the car decreases by 12.5% each year.
   a. Write an exponential decay model for the value of the computer.
   b. Use the model to estimate the value after 3 years.